Matematika 1 (2011/2012)

Linearni operatori

1. Jesu li sljedeća preslikavanja linearni operatori?

   a) $A : \mathbb{R}^3 \to \mathbb{R}^2 \quad A([x_1, x_2, x_3]) = [|x_1|, 0]$

   b) $A : \mathbb{R}^2 \to \mathbb{R} \quad A([x_1, x_2]) = x_1 \cdot x_2$

   c) $A : \mathbb{R}^3 \to \mathbb{R}^3 \quad A([x_1, x_2, x_3]) = [x_3, x_2, x_1]$

   d) $A : \mathbb{R}^3 \to \mathbb{R}^3 \quad A([x_1, x_2, x_3] = [2x_3 + x_1, 2x_1x_3, x_1 - x_2)$

   e) $A : \mathbb{R}^3 \to \mathbb{R}^3 \quad A([x_1, x_2, x_3] = [x_1, x_1 + x_2, x_1 - 2x_3]$

2. Nađite matricu operatora $A : \mathbb{R}^3 \to \mathbb{R}^3$ zadanog sa $A([x_1, x_2, x_3]) = [x_3, x_2, x_1]$ u kanonskoj bazi $\{[1, 0, 0], [0, 1, 0], [0, 0, 1]\}$.

3. Nađite matricu operatora $A : \mathbb{R}^3 \to \mathbb{R}^3$ zadanog sa $A([x_1, x_2, x_3] = [x_1, x_1 + x_2, x_2 + 3x_3)$ u bazi $\{[1, 1, 0], [0, 1, 1], [1, 0, 1]\}$.

4. Neka je $A : \mathcal{M}_{22} \to \mathcal{M}_{22}$ operator transponiranja, tj. $A(M) = M^T$. Dokažite da je to linearan operator i nađite mu matricu u kanonskoj bazi prostora $\mathcal{M}_{22}$:
$$\left\{ E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad E_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

5. Linearan operator $f : \mathbb{R}^2 \to \mathbb{R}^2$ u bazi $\{\vec{a}_1 = [1, 2], \vec{a}_2 = [1, 1]\}$ ima matrični zapis $\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$. Nađite mu matricu A' u bazi $\{\vec{b}_1 = [2, 3], \vec{b}_2 = [0, 1]\}$.

6. Vektor $\vec{x}$ u kanonskoj bazi $\{\vec{i}, \vec{j}, \vec{k}\}$ ima prikaz $\vec{x} = 6\vec{i} + 9\vec{j} + 14\vec{k}$. Odredite mu prikaz u bazi $\{\vec{i} + \vec{j} + \vec{k},\ \vec{i} + \vec{j} + 2\vec{k},\ \vec{i} + 2\vec{j} + 3\vec{k}\}$.

7. Odredite komponente vektora $\vec{x} = 2\vec{i} + 2\vec{j}$ i vektora $\vec{y} = \vec{i} + \sqrt{3}\vec{j}$ u novoj bazi $\{\vec{e}_1, \vec{e}_2\}$, gdje su $\vec{e}_1 = \frac{1}{2}\vec{i} + \frac{\sqrt{3}}{2}\vec{j},\ \vec{e}_2 = -\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{j}$.